SECTION A: STATISTICS

Answer ALL questions. Write your answers in the spaces provided.

1. Sara is investigating the variation in daily maximum gust, t kn, for Camborne in June and July 1987.

She used the large data set to select a sample of size 20 from the June and July data for 1987. Sara selected the first value using a random number from 1 to 4 and then selected every third value after that.

(a) State the sampling technique Sara used.

(1)

(b) From your knowledge of the large data set explain why this process may not generate a sample of size 20.

(1)

The data Sara collected are summarised as follows

$$n = 20 \qquad \sum t = 374 \qquad \sum t^2 = 7600$$

(c) Calculate the standard deviation.

(2)

(a) Sara used systematic sampling

(b) In the Large Data Set, there are some days with no recorded data, so this process may not generate a sample of size 20.

(c) $\sigma_t = \sqrt{\frac{\Sigma t^2}{n} - \bar{t}^2}$

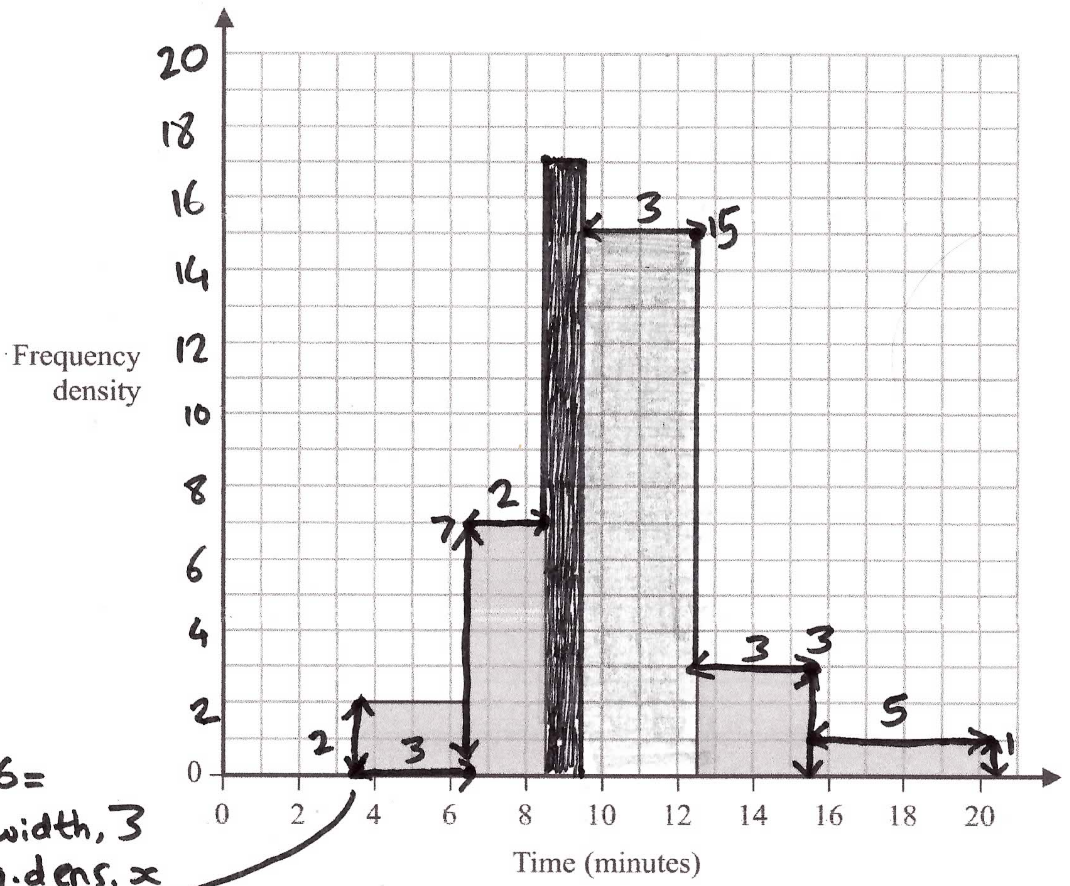
$\sigma_t = \sqrt{\frac{7600}{20} - \left(\frac{\Sigma t}{n}\right)^2}$

$\sigma_t = \sqrt{380 - \left(\frac{374}{20}\right)^2}$

$\sigma_t = \sqrt{380 - 18.7^2}$

$\boxed{\sigma_t = 5.51 \text{ (to 3s.f.)}}$

2. The partially completed histogram and the partially completed table show the time, to the nearest minute, that a random sample of motorists was delayed by roadworks on a stretch of motorway.



freq, 6= class width, 3 x freq.dens. x

6= 3x
x=2

Delay (minutes)	Number of motorists
4 – 6	6
7 – 8	14
9	17
10 – 12	45
13 – 15	9
16 – 20	5

Estimate the percentage of these motorists who were delayed by the roadworks for between 8.5 and 13.5 minutes.

(5)

frequency = class width x frequency density

Use the freq.density values to fill the table.

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Question 2 continued

Between 8.5 and 13.5 minutes:

$$\text{Frequency} = 17 + 45 + \left(\tfrac{1}{3} \times 9\right)$$

The class width is 3, but we only require the values for 1, so divide by 3

$$= 17 + 45 + 3$$

$$= \underline{65}$$

% of motorists between 8.5 and 13.5 minutes:

$$\% = \left(\frac{65}{6+14+17+45+9+5}\right) \times 100$$

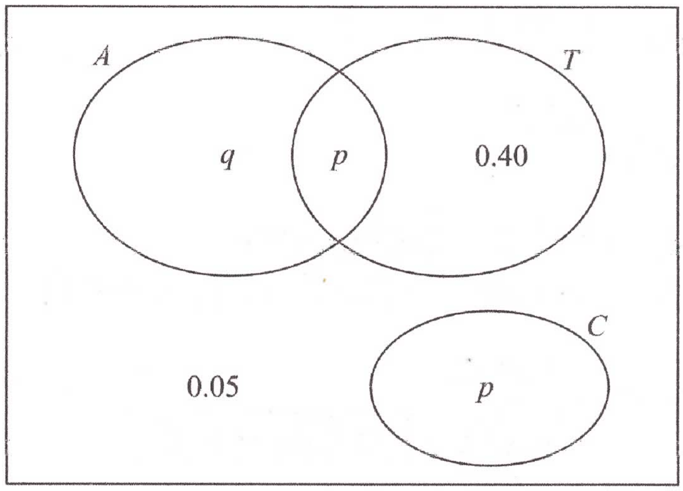
$$= \frac{65}{96} \times 100$$

$$= \boxed{67.7\% \text{ (to 3s.f.)}}$$

(Total for Question 2 is 5 marks)

3. The Venn diagram shows the probabilities for students at a college taking part in various sports.

A represents the event that a student takes part in Athletics.
T represents the event that a student takes part in Tennis.
C represents the event that a student takes part in Cricket.
p and q are probabilities.



The probability that a student selected at random takes part in Athletics or Tennis is 0.75

(a) Find the value of p.

(1)

(b) State, giving a reason, whether or not the events A and T are statistically independent. Show your working clearly.

(3)

(c) Find the probability that a student selected at random does not take part in Athletics or Cricket.

(1)

(a) $P(A \cup T) = 0.75$

$P(C) = 1 - (0.75 + 0.05)$

$= 1 - 0.8$

$= 0.2$

$\therefore \boxed{p = 0.2}$

Question 3 continued

(b) $P(A) = 0.35 \quad P(T) = 0.6$

$P(A \cap T) = 0.2$

$P(A) \times P(T) = 0.35 \times 0.6$

$= 0.21$

$0.2 \neq 0.21$, so events A and T are NOT statistically independent.

(c) $P(A' \cup C') = 0.4 + 0.05$

$= \boxed{0.45}$

(Total for Question 3 is 5 marks)

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4. Sara was studying the relationship between rainfall, r mm, and humidity, h%, in the UK. She takes a random sample of 11 days from May 1987 for Leuchars from the large data set.

She obtained the following results.

h	93	86	95	97	86	94	97	97	87	97	86
r	1.1	0.3	3.7	20.6	0	0	2.4	1.1	0.1	0.9	0.1

Sara examined the rainfall figures and found

$$Q_1 = 0.1 \qquad Q_2 = 0.9 \qquad Q_3 = 2.4$$

A value that is more than 1.5 times the interquartile range (IQR) above Q_3 is called an outlier.

(a) Show that $r = 20.6$ is an outlier.

(1)

(b) Give a reason why Sara might:

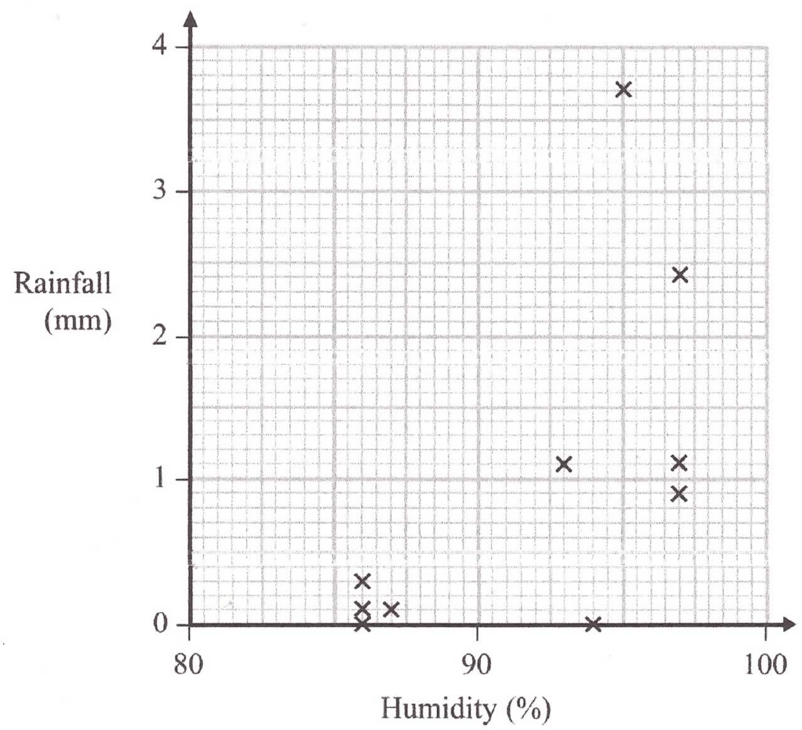
(i) include

(ii) exclude

this day's reading.

(2)

Sara decided to exclude this day's reading and drew the following scatter diagram for the remaining 10 days' values of r and h.



(c) Give an interpretation of the correlation between rainfall and humidity.

(1)

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Question 4 continued

The equation of the regression line of r on h for these 10 days is $r = -12.8 + 0.15h$

(d) Give an interpretation of the gradient of this regression line.

(1)

(e) (i) Comment on the suitability of Sara's sampling method for this study.

(ii) Suggest how Sara could make better use of the large data set for her study.

(2)

(a) $IQR = Q_3 - Q = 2.4 - 0.1 = 2.3$

$2.4 + 1.5 \times 2.3 = 5.85$

$r = 20.6 > 5.85$, so $r = 20.6$ is an outlier

(b) (i) She should include it because it is a piece of data and all data should be considered.

(ii) She could exclude it since it is an extreme value and affect the investigation.

(c) As humidity increases, rainfall increases.

(d) The gradient (0.15) represents that there's a 0.15mm increase in rainfall per percentage of humidity.

(e) (i) Sara's sampling method isn't very good since she only uses 11 days out of the whole month, and only uses one specific location.

(ii) Sara could use data from more than one UK location and also use a wider range of months with more days per month.

(Total for Question 4 is 7 marks)

5. (a) The discrete random variable $X \sim B(40, 0.27)$

Find $P(X \geqslant 16)$

(2)

Past records suggest that 30% of customers who buy baked beans from a large supermarket buy them in single tins. A new manager suspects that there has been a change in the proportion of customers who buy baked beans in single tins. A random sample of 20 customers who had bought baked beans was taken.

(b) Write down the hypotheses that should be used to test the manager's suspicion.

(1)

(c) Using a 10% level of significance, find the critical region for a two-tailed test to answer the manager's suspicion. You should state the probability of rejection in each tail, which should be less than 0.05

(3)

(d) Find the actual significance level of a test based on your critical region from part (c).

(1)

One afternoon the manager observes that 12 of the 20 customers who bought baked beans, bought their beans in single tins.

(e) Comment on the manager's suspicion in the light of this observation.

(1)

Later it was discovered that the local scout group visited the supermarket that afternoon to buy food for their camping trip.

(f) Comment on the validity of the model used to obtain the answer to part (e), giving a reason for your answer.

(1)

(a) $P(X \geq 16) = 1 - P(X \leq 15)$

$= 1 - 0.9491$

$= \boxed{0.0509 \quad (\text{to } 4\text{d.p.})}$

(b) $H_0: p = 0.3$, $H_1: p \neq 0.3$

(c) Let Y be the number of cutsomers who buy tins out of a sample of 20.

$Y \sim B(20, 0.3)$

$P(Y \leq 3) = \underline{0.1071}$ > 0.05
$P(Y \leq 2) = \underline{0.0355}$ < 0.05

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Question 5 continued

Critical region for the lower tail is $y \leq 2$

$P(X \geq 9) = 1 - P(X \leq 8)$
$= 1 - 0.8867$
$= 0.1133 \quad > 0.05$

$P(Y \geq 10) = 1 - P(Y \leq 9)$
$= 1 - 0.9520$
$= 0.0480 \quad < 0.05$

Critical region for the upper tail is $y \geq 10$

$\therefore$ $\boxed{\text{Critical region is } y \leq 2 \text{ or } Y \geq 10}$

(d) Actual significance level $= 0.0355 + 0.0480$

$\boxed{= 0.0835}$

(e) Since the observed value of 12 lies within the critical region, there is sufficient evidence to reject H_0: the manager's belief is supported.

(f) If the visitors came as a scout group, this means that it wasn't a random sample taken and also each tin of baked beans wasn't bought independently. This invalidates the use of a binomial distribution as a model and thus the answer in part (e) too.

(Total for Question 5 is 9 marks)

TOTAL FOR SECTION A IS 30 MARKS

SECTION B: MECHANICS

Answer ALL questions. Write your answers in the spaces provided.

Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8\text{ m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

6.

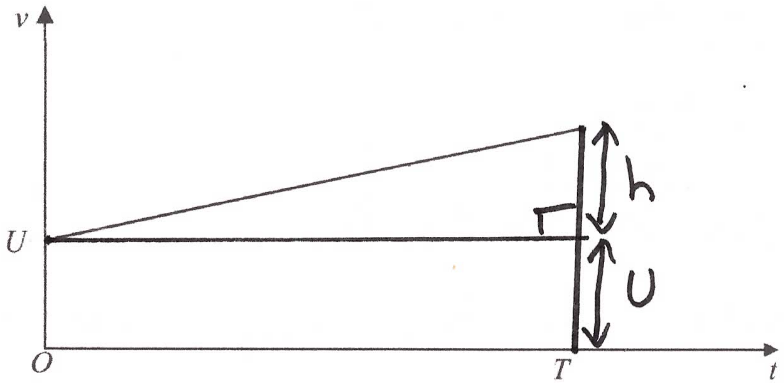


Figure 1

A car moves along a straight horizontal road. At time $t = 0$, the velocity of the car is $U\text{ m s}^{-1}$. The car then accelerates with constant acceleration $a\text{ m s}^{-2}$ for T seconds. The car travels a distance D metres during these T seconds.

Figure 1 shows the velocity-time graph for the motion of the car for $0 \leqslant t \leqslant T$.

Using the graph, show that $D = UT + \frac{1}{2}aT^2$.

(No credit will be given for answers which use any of the kinematics (*suvat*) formulae listed under Mechanics in the AS Mathematics section of the formulae booklet.)

(4)

Distance, D = Area under graph

$$= UT + \frac{1}{2}hT$$

Since the gradient is the acceleration, $h = aT$

So, $D = UT + \frac{1}{2}(aT)T$

$$\therefore \boxed{D = UT + \frac{1}{2}aT^2}$$

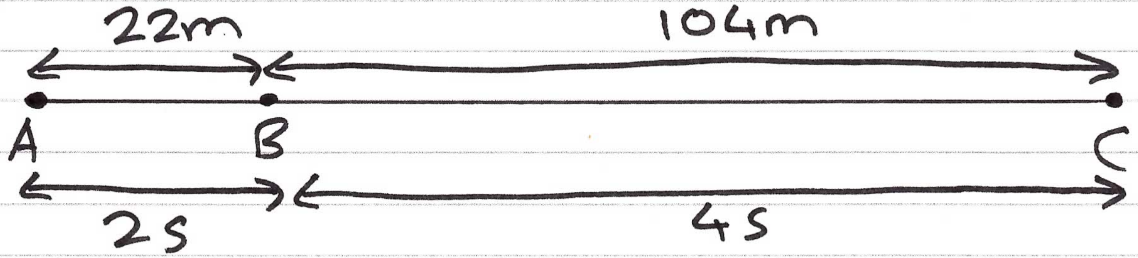
7. A car is moving along a straight horizontal road with constant acceleration. There are three points A, B and C, in that order, on the road, where $AB = 22$ m and $BC = 104$ m. The car takes 2 s to travel from A to B and 4 s to travel from B to C.

Find

(i) the acceleration of the car,

(ii) the speed of the car at the instant it passes A.

(7)



(i) $s = ut + \frac{1}{2}at^2$

$22 = u(2) + \frac{1}{2}a(2)^2$

$22 = 2u + 2a$ ①

Also, $126 = u(6) + \frac{1}{2}a(6)^2$

$126 = 6u + 18a$ ②

To find a, eliminate u by solving simultaneous equations:

$2u + 2a = 22$ ① $\times 3$ $6u + 6a = 66$
$6u + 18a = 126$ ② $\times 1$ $6u + 18a = 126$ −

$-12a = -60$

$\therefore a = 5$

$\boxed{\text{acceleration of the car} = 5\text{ms}^{-2}}$

Question 7 continued

(ii) Substitute a into ① for u:

$$2u + 2a = 22$$

$$2u + 2(5) = 22$$

$$2u + 10 = 22$$

$$2u = 12$$

$$\therefore u = 6$$

Speed of car as it passes $A = 6\text{ms}^{-1}$

(Total for Question 7 is 7 marks)

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8. A bird leaves its nest at time $t = 0$ for a short flight along a straight line.

The bird then returns to its nest.

The bird is modelled as a particle moving in a straight horizontal line.

The distance, s metres, of the bird from its nest at time t seconds is given by

$$s = \frac{1}{10}(t^4 - 20t^3 + 100t^2), \quad \text{where } 0 \leqslant t \leqslant 10$$

(a) Explain the restriction, $0 \leqslant t \leqslant 10$

(3)

(b) Find the distance of the bird from the nest when the bird first comes to instantaneous rest.

(6)

(a) When $t=0$, $s = \frac{1}{10}(0^4 - 20(0)^3 + 100(0)^2)$

$\underline{s=0}$

When $t = 10$, $s = \frac{1}{10}(10^4 - 20(10)^3 + 100(10)^2)$

$= \frac{1}{10}(10000 - 20(1000) + 100(100))$

$= \frac{1}{10}(10000 - 20000 + 10000)$

$\underline{= 0}$

So when $t=0$ and $t=10$, $s=0$

Now, $s = \frac{1}{10}(t^4 - 20t^3 + 100t^2)$

$s = \frac{1}{10}t^2(t^2 - 20t + 100)$

$s = \frac{1}{10}t^2(t-10)(t-10)$

Question 8 continued

$$s = \frac{1}{10} t^2 (t-10)^2$$

$\therefore$ s is a perfect square, so
$s > 0$ for $0 < t < 10$

(b) Differentiate displacement, s for velocity, v:

$$v = \frac{1}{10}(4t^3 - 60t^2 + 200t)$$

When the bird reaches rest, velocity $= 0\text{ms}^{-1}$

$$\therefore \frac{1}{10}(4t^3 - 60t^2 + 200t) = 0$$

$$\frac{1}{10} t(4t^2 - 60t + 200) = 0$$

$$\frac{4}{10} t(t^2 - 15t + 50) = 0$$

$$\frac{2}{5} t(t-10)(t-5) = 0$$

Either $t=0$ or $t=5$ or $t=10$

When the bird first reaches instantaneous rest, $t=5$. It isn't $t=0$, because this is when the bird leaves the nest.

(Total for Question 8 is 9 marks)

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Question 8 continued

Now, substitute $t=5$ into

$$s = \frac{1}{10}(t^4 - 20t^3 + 100t^2):$$

$$s = \frac{1}{10}(5^4 - 20(5)^3 + 100(5)^2)$$

$$s = \frac{1}{10}(625 - 20(125) + 100(25))$$

$$s = \frac{1}{10}(625 - 2500 + 2500)$$

$$s = \frac{1}{10}(625)$$

$$\boxed{s = 62.5 \text{ m}}$$

distance of the bird from the nest when it first comes to instantaneous rest is 62.5m

(Total for Question 8 is 9 marks)

9.

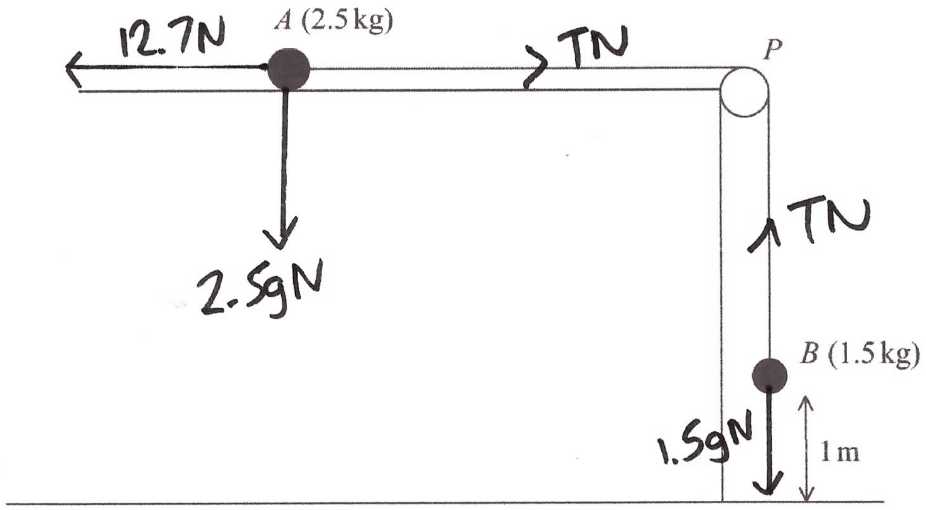

Figure 2

A small ball A of mass 2.5 kg is held at rest on a rough horizontal table.

The ball is attached to one end of a string.

The string passes over a pulley P which is fixed at the edge of the table. The other end of the string is attached to a small ball B of mass 1.5 kg hanging freely, vertically below P and with B at a height of 1 m above the horizontal floor.

The system is release from rest, with the string taut, as shown in Figure 2.

The resistance to the motion of A from the rough table is modelled as having constant magnitude 12.7 N. Ball B reaches the floor before ball A reaches the pulley.

The balls are modelled as particles, the string is modelled as being light and inextensible, the pulley is modelled as being small and smooth and the acceleration due to gravity, g, is modelled as being $9.8\,\text{m s}^{-2}$.

(a) (i) Write down an equation of motion for A.

(ii) Write down an equation of motion for B.

(4)

(b) Hence find the acceleration of B.

(2)

(c) Using the model, find the time it takes, from release, for B to reach the floor.

(2)

(d) Suggest two improvements that could be made in the model.

(2)

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Question 9 continued

(a) (i) Resolving ($\rightarrow$):

$$\underline{T - 12.7 = 2.5a} \quad ①$$

Resultant Force = mass × acceleration, so $F_{Res} = 2.5a$

(ii) Resolving ($\downarrow$):

$$\underline{1.5g - T = 1.5a} \quad ②$$

$F_{Res} = ma$
$= 1.5a$

(b) ① + ②: $2 = 4a$

$$\therefore \underline{a = 0.5\,\text{ms}^{-2}}$$

(c) $s = ut + \frac{1}{2}at^2$

For B, $u = 0$, so $ut = 0$

$$\therefore s = \frac{1}{2}at^2$$

Distance travelled, s is known to be 1m:

$$\text{so, } 1 = \frac{1}{2}(0.5)t^2$$

$$1 = 0.25t^2$$

$$4 = t^2$$

$\boxed{t = 2 \text{ seconds}}$ (since time is positive)

(d) • Improve the accuracy of the value of g.
• Include dimensions of the ball so that we can get an exact distance that it falls.